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Optimal vs. Traditional Securities under Moral Hazard

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Abstract

This paper provides an explanation for the widespread use of traditional securities by well-established firms. Standard moral hazard models predict that equity, debt, and warrants are almost never optimal financing instruments. I show that issuing these securities is, nevertheless, nearly optimal: the issuer would gain very little by using non-traditional securities instead. Combined with equity, one debt issue (without multiple layers of seniority) and one warrant issue (without multiple exercise prices) suffice to achieve near optimality. The near optimality of traditional financing depends crucially on the issuer's ability to use warrants in addition to debt and equity.

I. Introduction

Well-established firms raise external capital mainly through traditional financial securities: debt and equity, possibly bundled with warrants. Agency models are natural candidates to analyze this financing decision. Yet, standard moral hazard models predict that traditional securities are not optimal. The issuer increases his expected payoff by selling securities whose returns are *not* simple piece-wise linear functions of cash flows.

This apparent discrepancy between theory and corporate practice might indicate that basic agency models do not properly represent the conditions faced by such firms; however, design-related transaction costs may be lower for securities with piece-wise linear returns. This may explain why simple instruments like debt (to reduce agency problems) and equity (for risk sharing) have been used for centuries.[1] Moreover, investors are typically well versed in the characteristics of debt, equity, and warrants, but have to expend resources to become

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[1]See Allen and Gale ((1994), chapter 2) and the references cited therein.

acquainted with unfamiliar securities. Gale (1992), for example, claims that securities with atypical legal provisions are hard for investors to evaluate because they have not been tested in court. More importantly, markets for non-standard securities may not be as liquid as those for traditional financial instruments. Empirical evidence from Amihud and Mendelson (1986), (1991) suggests that lower liquidity is costly to the issuer. These differential transaction costs, associated with the issue or purchase of non-traditional securities, imply that issuing traditional financial instruments could well be optimal—provided there are not sizable welfare or efficiency losses.

This paper quantifies the deadweight costs that result from limiting the firm to equity, a single straight debt issue, and a single warrant issue in a standard moral hazard framework in which combinations of these three instruments are not optimal for the issuer. Across a wide range of common parameterizations, I show that these costs are tiny (usually 0.01% of the amount invested) and that traditional securities are nearly optimal. I conclude from these results that the issuer's gains from non-standard financing contracts are, in practice, likely to be very small.

I derive these findings by building a simple, one-period model of investment financing under moral hazard. The investment's return, which is publicly observable at no cost, depends on the managerial effort level and a random production shock. The risk-averse owner-manager designs the financing package to maximize his total expected utility, subject to limited liability, to incentive compatibility, and to the constraint that investors earn a competitive expected rate of return. Except under special conditions, the optimal financing package never corresponds to a combination of traditional financial instruments.

To show that a simple traditional capital structure is nevertheless almost optimal, I characterize the financing problem when outside investors can be offered a maximum of three securities: equity, straight debt, and warrants (the owner-manager can earn a base salary and receive a residual equity stake and some of the warrants). I contrast the deadweight costs caused by this additional contracting restriction to the losses due solely to incentive compatibility requirements. That comparison provides a natural benchmark from which to assess the costs of more restricted contracting, relative to the costs of contracting per se. Because the equations that characterize the optimal and traditional financing contracts cannot be solved analytically, I carry out the comparison by parameterizing the model and solving it numerically.

Moral hazard itself is very costly in this environment. Even with the optimal (but non-traditional) financing package, the owner-manager's certainty equivalent consumption under moral hazard is typically two-thirds less than it would be without incentive compatibility problems. The extent of that deadweight cost, equivalent to almost 12% of the resources invested, is consistent with the large empirical estimates of Ferrall and Shearer (1994). The first major result is that the extra deadweight loss from limiting security offerings to equity, one straight debt issue (without multiple layers of seniority), and one warrant issue (without multiple exercise prices) is, in contrast, very small. It never exceeds a mere 0.15%, and typically amounts to a minuscule 0.01%, of the amount invested. These numbers are negligible, given that the calibrated rate of return on assets is 12.1%.

The second major result is that the near optimality of traditional financing depends crucially on the issuer's ability to float warrants together with debt and equity. Ruling out all securities other than equity, by comparison, always gives rise to large additional welfare costs. For many parameterizations, investment is unprofitable under pure equity financing, even though it would have been profitable under the (non-traditional) optimal financing contract. In many cases, adding unsubordinated straight debt reduces the deadweight costs of the exogenous contracting restrictions by as much as 0.34% of the amount invested. Still, there are cases for which investment is unprofitable under debt and equity financing. Adding a single warrant issue, with a unique exercise price, makes these investments profitable. Indeed, for all reasonable parameterizations such that investment would take place under non-traditional financing, there exists a simple combination of equity, straight debt, and warrants that makes investing optimal as well. Furthermore, the resulting contract exhibits welfare and efficiency properties almost identical to those of its non-traditional counterpart.

The next section summarizes the related research. Section III describes the setup, while Section IV characterizes the financing problem under different contracting regimes. Section V parameterizes the model. Section VI computes the losses of firm value and expected managerial utility brought about by the various contracting restrictions. Section VII demonstrates the importance of debt and warrants. Section VIII discusses the source and robustness of the results. Section IX concludes.

II. Related Work

Many papers share with this one the motivation of rationalizing the prevalence of simple, traditional securities. The strategy adopted in most of these papers, however, is to construct environments in which such securities can indeed be shown to be optimal. Williams (1988), for example, uses a costly state verification framework to show that a risk-neutral entrepreneur will optimally use debt and equity when informational asymmetries about cash flows are monitorable. Chiesa (1992) instead assumes that cash flows can be contracted on but that the entrepreneur's actions cannot. When the entrepreneur is risk neutral and a random state of nature is determined prior to his effort choice, Chiesa shows that a debt contract plus a series of warrants for the lender and options for the entrepreneur is optimal.[2]

My approach is different in that I focus on the question of how suboptimal traditional securities are in a more typical contracting setting in which the optimal financing contract is likely to be very complicated. In such an environment, I show quantitatively that even small transaction costs associated with non-traditional securities make a simple, traditional capital structure optimal.[3]

[2]Like the present paper, both Williams (1988) and Chiesa (1992) abstract from issues of adverse selection, investor diversity, takeovers and replacement of managers, and impossibilities to precommit to a course of action over a number of periods. For reviews of optimal financial contracting under various such circumstances, see Allen and Winton (1994) and Fluck (1998).

[3]More generally, my results, therefore, help explain why simple piece-wise linear sharing rules are ubiquitous. Again, unlike previous studies (Ross (1974), Dybvig and Spatt (1986), Gjesdal (1988),

This paper is, therefore, related to Bhattacharya and Pfleiderer (1985), Stoughton (1993), and Boyd and Smith (1994), in that I measure costs associated with the approximate implementation of some optimal financial outcomes. Thus, like these papers, I depart from the optimal security design approach of Allen and Gale (1988)—even though the transaction costs that limit the diversity of securities in that full information model are similar in spirit to the costs associated with the issue or purchase of exotic securities that make traditional financing optimal in my agency framework.

Bhattacharya and Pfleiderer (1985) and Stoughton (1993) model a risk-averse investor who wishes to allocate her wealth between two assets. After either screening potential portfolio managers (Bhattacharya and Pfleiderer) or eliciting research effort from a given analyst (Stoughton), she must motivate the hired professional to truthfully report his payoff-relevant information. In both setups, a quadratic contract induces truthfulness at the cost of second-best risk sharing between principal and agent.[4] That cost is negligible, and the investor's expected utility is nearly first-best, when the investor is almost risk neutral and is large relative to her agent. I focus instead on a situation in which, even though investors are risk neutral, the optimal contract is far from first-best and moral hazard itself is very costly. In such an environment, I show that a limited menu of traditional securities helps control agency costs almost as effectively as complicated optimally-designed securities.

Boyd and Smith (1994) use a general equilibrium, costly state verification framework to show numerically that the welfare costs from restricting corporate financing to standard debt and internal equity are minor. My results agree with theirs, since I also show that the deadweight losses from exogenous constraints on admissible contracts are very small. My goal, however, is different. First, outside equity and warrants have no value in their framework because outside investors never observe the actual earnings, there is no ex-ante monitoring, and managers can treat all unused corporate funds as their own income. Second, they model the borrowing practices of large corporations and, hence, assume that both lenders and borrowers are risk neutral. I tackle the financing decisions of firms whose managers hold a significant share of their company's stock and take actions that significantly affect the distribution of earnings. Because these individuals are unlikely to hold well-diversified portfolios, I assume that they are risk averse. The difference between my results and those of Innes (1990) confirms the importance of such risk-sharing considerations. In a framework similar to mine, Innes (1990) shows that straight debt financing is optimal for risk-neutral entrepreneurs. Here, equity and warrants are also necessary to mitigate agency problems between outside investors and risk-averse insiders.

Sung (1995)), I do not attempt to identify conditions that make such sharing rules optimal under moral hazard. Instead, I demonstrate numerically that, in a standard moral hazard framework, very small deadweight costs arise from exogenously restricting the contracting space to simple (three-piece, two-kink) piece-wise linear functions of output.

[4]Intuitively, as long as asset returns are symmetrically distributed, a quadratic contract reduces the agent's temptation to use portfolio selection to undo either a misrepresentation of his type (Bhattacharya and Pfleiderer) or his effort choice (Stoughton). In my standard agency setup, the agent's action and disutility occur simultaneously: hence, except in special cases, there is no presumption that a quadratic contract would be remotely optimal.

The near optimality of my simple capital structure presents an interesting counterpoint to studies of the equity premium puzzle by Telmer (1993) and Heaton and Lucas (1996). These authors show numerically that, in dynamic endowment economies, market completeness is not quantitatively important because trading risk-free bonds (and, possibly, equity) allows individuals to share most of their undiversifiable labor income risk. Baxter and Crucini (1995) find similar results in a production economy. Unlike those papers, I model the tradeoff between productive efficiency and risk sharing under moral hazard. In such an environment, I show that banning warrants altogether would imply large deadweight losses.

The prediction that issuing warrants can significantly increase firm value is consistent with the evidence documented by Conrad (1989) and Detemple and Jorion (1990), who show empirically that the introduction of individual options improves the risk-return tradeoff for the underlying stocks. It is also in line with results by Green (1984) and Chiesa (1992) that bundling warrants together with debt can alleviate moral hazard. The first-best outcome cannot be attained in my setup because of risk aversion and limited liability. Yet, a simple combination of warrants, debt, and equity mitigates agency conflicts and makes up a nearly optimal financing package. In contrast to Green (1984), warrants here are a complement—rather than a substitute—to equity. My results generalize those of Chiesa (1992), whose solution entails multiple warrant issues, by introducing risk aversion and, more importantly, by showing numerically that a single warrant issue with a unique exercise price is enough to achieve near optimality.

III. The Model

Consider a one-period model of investment financing under moral hazard. At time 0, a company run by its owner-manager needs a fixed amount I of outside capital to fund a new project. The project's intrinsic quality is common knowledge but its uncertain time-1 return, $y \in \Re^+$, depends on the owner-manager's unobservable effort level, $a \in \Re^+$. The cash flow y can be viewed as a random variable with density function $f(y, a)$ parameterized by a. The support of y is independent of a.

The time-1 return y constitutes the only potential source of income for the owner-manager. His utility is a separable function of his income, c, and time-0 effort, $a : U(c, a) \equiv u(c) - v(a)$. The functions $u(\cdot) : \Re \rightarrow \Re$ and $v(\cdot) : \Re^+ \rightarrow \Re^+$ are thrice continuously differentiable, with $u(\cdot)$ strictly increasing concave and $v(\cdot)$ increasing convex. Unlike the owner-manager, outside investors hold well-diversified portfolios and are indifferent to firm-specific risk. I approximate this fact by positing that investors are risk neutral. For simplicity, I assume a zero discount rate.

The owner-manager designs the financing package to maximize his total expected utility, subject to three constraints. The first is limited liability (LL). A straightforward reason for imposing this constraint is that, since the owner-manager's only source of income is y, violations of his liability limits would imply negative managerial consumption.[5]

[5]For some common preference choices (e.g., exponential utility) negative consumption is in fact well defined and, in the present environment, can be interpreted in terms of debtors' prison (Robe

The second contracting constraint is incentive compatibility (IC). Because the owner-manager's effort level a is unobservable, the contract must ensure that he finds it optimal to work as promised given the financing terms. I impose the weaker but mathematically more tractable requirement that he choose an effort level at which his utility is at a stationary point. I then verify numerically that this first-order approach, which is likely to be legitimate in this model under some restrictions on $f(\cdot)$ and $u(\cdot)$ identified by Jewitt (1988), does ensure incentive compatibility.[6]

The third constraint is participation (IR). Outside investors must earn at least a competitive expected rate of return on I (their investment). Without loss of generality, this rate is set equal to 0.

Markets are otherwise perfect. In particular, there are no taxes or bankruptcy costs; all flows of funds are costlessly verifiable; and the statistical properties of the stochastic variable y, the production process $f(\cdot)$, and the preferences of all players, are common knowledge.

IV. Financing Contracts

Let $t(y)$ denote the owner-manager's monetary payoff as a function of the project's return, y. In a "first-best" world where a is costlessly observable, the risk-averse manager would sell off the project to risk-neutral investors for a fixed price. Given that a is unobservable, the owner-manager faces the "second-best" problem,

$$\text{(1)} \qquad \max_{a,t(\cdot)} \left[\int_0^\infty u(t(y))f(y,a)dy - v(a) \right]$$

$$\text{s.t.} \quad \text{(LL)} \quad 0 \le t(y) \le y,$$

$$\text{(IC)} \quad \int_0^\infty u(t(y))f_a(y,a)dy - v'(a) = 0,$$

$$\text{(IR)} \quad \int_0^\infty [y - t(y)]f(y,a)dy = I.$$

Let λ and μ be the Lagrange multipliers for the participation and incentive compatibility constraints, respectively. When the owner-manager does not have

(1998), see also Welch (1995)). An alternative justification for imposing limited liability, therefore, is that I assume away debtors' prison. This constraint is reasonable, given that jailing debtors for non-fraudulent default on commercial debt is an obsolete practice. The constraint is usually binding. Conditional on the liability regime, however, my main results are robust to the removal of liability limits and to the introduction of debtors' prison—see Section VIII.D.

[6]The gamma, Poisson, and chi-squared distributions, among others, satisfy the conditions on $f(\cdot)$. Restrictions on preferences are met by any constant absolute risk-averse utility, and by any non-decreasing relative risk-averse utility with coefficient of relative risk aversion strictly greater than one-half. Jewitt (1988) shows that these conditions validate the first-order approach in the standard agency model of Holmström (1979). In the absence of liability limits, solutions to that model and to the present one characterize points on the same utility possibility frontier. Under the Jewitt conditions, the approach would, therefore, remain legitimate in my setup. My owner-manager, however, has limited liability and can also face other exogenous restrictions on security design. As a result, the Jewitt restrictions might not suffice for the first-order approach to be valid.

limited liability, his monetary payoff is strictly increasing in cash flows under the Jewitt (1988) restrictions. Once liability limits are imposed, the monetary payoff that solves program (1), $t^*(y)$, is, therefore, given by

$$\text{(2)} \qquad \text{either } 0 = t^*(y) \qquad \text{or} \qquad \frac{1}{u'(t^*(y))} = \overline{\lambda} + \overline{\mu}\frac{f_a(y,a)}{f(y,a)},$$

where a, $\overline{\lambda} \equiv 1/\lambda$ and $\overline{\mu} \equiv \mu/\lambda$ solve the first-order conditions of program (1).

Innes (1990) shows that if the owner-manager were risk neutral then he could optimally use debt financing. Intuitively, debt is the monotonic contract that maximizes the owner-manager's incentives to strive for high cash flows by leaving him nothing in low cash flow states.

In my setup, productive efficiency must be traded off against risk sharing. Condition (2) suggests that, as a result, the optimal contract almost never involves traditional securities. To see this, recall that debt, equity, and warrants all leave the owner-manager with monetary returns $t(y)$ that are piece-wise linear in cash flows. With logarithmic utility and exponentially distributed cash flows, the solution $t^*(y)$ to condition (2) is a two-segment function that corresponds to debt and equity financing. For more general preferences and technologies, however, $t^*(y)$ is not piece-wise linear and differs from a combination of traditional financial instruments. The Appendix provides a proof for the preferences and technologies used in the computations.

In practice, however, well-established companies of all sizes raise external capital chiefly by selling traditional securities like debt, equity, and warrants.[7] Clearly, if equity can be supplemented by sufficiently many debt and warrants issues with suitably different seniority levels and exercise prices, then some combination of traditional securities must be very close to the second-best contract.[8] The key question, then, is whether even a *simple* capital structure—consisting only of equity, unsubordinated straight debt, and a single warrant issue—allows the issuer to achieve expected utility levels that are extremely close to their second-best counterpart.

Requiring that the financing contract comprise only these three traditional instruments simplifies the owner-manager's problem. He merely needs to find the amount of straight debt to issue, the fraction of the firm's equity rights to sell, and the exercise price for the warrants (if any). Formally, when restricted to using equity, one straight debt issue (without multiple layers of seniority), and one class

[7] For instance, Petersen and Rajan (1994) document that, in a sample of 3,404 U.S. firms answering the 1988 National Survey of Small Business Finances, 91% of the companies with book value of assets greater than $488,000 made use of debt financing, with 62% of the total borrowed from banks. Helwege and Liang (1996) report that firms that had completed their initial public offering in 1983 accessed public capital markets in the following decade by issuing common equity, private debt, and straight and convertible bonds.

[8] To see this, notice that the closure of the set of continuous functions that consist of finite or many countable linear pieces is the set of continuous functions—see Royden ((1988), p. 50). Hence, given that the second-best contract is a.e. continuous in cash flows on the whole support, it can be approximated arbitrarily well by a continuous piece-wise linear function. Thus, as the number of debt and warrant issues increases, the number of linear pieces increases and the traditional contract must converge to the second-best contract.

of warrants (with a single exercise price), the owner-manager faces the following third-best problem,

$$(3) \quad \max_{\substack{a \\ \alpha_1,\alpha_2,\alpha_3 \\ \beta, X_1, X_2}} \left[\int_0^{X_1} u(T_1(y))f(y,a)dy + \int_{X_1}^{X_2} u(T_2(y))f(y,a)dy + \int_{X_2}^{\infty} U(T_3(y))f(y,a)dy - v(a)\right]$$

$$\text{s.t.} \quad \text{(LL)} \quad 0 \le T_i(y) \le y \quad \forall y \quad (i = 1,2,3),$$

$$\text{(IC)} \quad \frac{\partial}{\partial a}\left[\int_0^{X_1} u(T_1(y))f(y,a)dy + \int_{X_1}^{X_2} u(T_2(y))f(y,a)dy + \int_{X_2}^{\infty} u(T_3(y))f(y,a)dy - v(a)\right] = 0,$$

$$\text{(IR)} \quad \int_0^{X_1} (y - T_1(y))f(y,a)dy + \int_{X_1}^{X_2} (y - T_2(y))f(y,a)dy + \int_{X_2}^{\infty} (y - T_3(y))f(y,a)dy = I,$$

with $T_1(y) = \beta + \alpha_1 y$; $T_2(y) = \beta + [\alpha_1 X_1 + \alpha_2(y - X_1)]$; $T_3(y) = \beta + [\alpha_1 X_1 + \alpha_2(X_2 - X_1) + \alpha_3(y - X_2)]$. The simple linear functions $T_1(\cdot), T_2(\cdot)$, and $T_3(\cdot)$ determine the share of cash flows that accrues to the owner-manager when y falls in successively higher ranges: $[0, X_1], [X_1, X_2]$, and $[X_2, \infty]$. The premium, β, that outside investors may pay in addition to their investment, I, can alternatively be viewed as a constant salary given to the owner-manager on top of his state-contingent return.

Pure equity financing corresponds to $X_1 = X_2 = 0$; straight debt and equity financing obtains when $X_1 = \alpha_2 = 0$ and $0 \le X_2 < \infty$ (X_2 is the promised debt repayment). When $0 < \alpha_3 < \alpha_2 \le 1$ and $X_1 = 0$, the contract involves i) internal equity and ii) external equity plus one class of warrants with total exercise price $X_2(\alpha_2 - \alpha_3/\alpha_2)$. If $0 < \alpha_2 < \alpha_3 \le 1$ and $X_1 = 0$, the contract again entails internal and external equity, but it is now the owner-manager who holds the entire warrant issue—with total exercise price $X_2(\alpha_3 - \alpha_2)/(1 - \alpha_2)$.

V. Parameterization

To quantify whether issuing a few traditional securities rather than optimally-designed financial instruments creates sizable deadweight losses, one must parameterize the model, solve it numerically, and carry out robustness checks. Numerical solutions are necessary for two reasons. First, the second-best and third-best contracts are found by solving the first-order conditions of programs (1) and

(3), respectively. Analytical solutions exist in trivial cases only. Second, as argued in Section III, one must verify numerically that the solutions to programs (1) and (3) are incentive compatible.

A. Choice of Functional Forms

Computations require functional forms for the technology and for the owner-manager's utility function, $U(c,a) \equiv u(c) - v(a)$. Assume that he has convex power disutility from effort,

$$v(a) \equiv \frac{a^n}{A}, \quad n > 1, \tag{4}$$

where $A > 0$ is a scaling factor. Varying A and n provides two degrees of freedom, which makes disutility function (4) general enough for numerical computations. Furthermore, as long as $v(\cdot)$ is convex, it follows from condition (2) that the functional form of the second-best contract is independent of $v(\cdot)$. In that sense, positing (4) entails no loss of generality.

I choose the technology and the owner-manager's preferences over consumption to meet the Jewitt (1988) conditions. In this and the next two sections, I posit that the owner-manager has negative exponential utility over consumption with coefficient of absolute risk aversion $\gamma > 0$,

$$u(c) \equiv -e^{-\gamma c}, \tag{5}$$

and that cash flows follow a gamma distribution with parameters τ and $\tau/(a\theta)$,

$$f(y,a) \equiv \frac{y^{\tau-1}e^{\frac{-y\tau}{a\theta}}}{\left(\frac{a\theta}{\tau}\right)^{\tau}(\tau-1)!}, \qquad \theta > 0, \quad \tau \in 1N. \tag{6}$$

Preferences (5) are widely used in the finance and agency literatures (e.g., Bhattacharya and Pfleiderer (1985), Stoughton (1993), Sung (1995)). My main conclusions are not affected if I use constant relative risk-averse utility under which the first-order approach holds—see Section VIII.

Technology (6) offers two benefits. First, some other common technologies are special cases. Letting $\tau = 1$, for example, would yield an exponential distribution. In Sections VI and VII, $\tau \geq 2$ is set to generate a unimodal, hump-shaped distribution. Section VIII illustrates that my main results are robust to the particular value taken by the shape parameter τ. Accordingly, I focus on $\tau = 2$ in order to fully exploit the second advantage of technology (6): with functional forms (5) and (6), all the integrals that appear in the first-order conditions of programs (1) and (3) have a closed-form solution that becomes more complex as τ increases. The same is true for the Hessian of program (3). Setting $\tau = 2$ significantly improves computational tractability, which is central to the precision of my solutions.

B. Choice of Parameter Values

With the chosen functional forms, the model has several free parameters, including γ, n, A, θ, and the impact of managerial action on firm performance. I calibrate these parameters with U.S. data, under the assumption that the second-best environment is an appropriate representation of the conditions faced by owner-managers when they determine their firms' optimal financing.

The impact of managerial action on firm performance does not have a good empirical estimate. For large firms, Haubrich (1994) argues that assuming an expected firm value increase of 4% when the CEO takes the highest possible action rather than the lowest one seems consistent with the evidence on CEO turnover given by Weisbach (1988).[9] This productivity measure must be adjusted, however, because Haubrich uses a two-point distribution of cash flows—both possible values of which are strictly positive. In contrast, with production function (6), very low levels of managerial effort yield almost zero expected cash flows. For the type of firms studied in this paper, managerial effort is likely crucial to the survival of the company. Accordingly, I require that the marginal managerial productivity *at the optimum* lie in the range presented by Haubrich.[10] The percentage marginal productivity of effort, under technology (6), is given by

$$\text{(7)} \qquad \frac{\partial E[y|a,\theta]/\partial a}{E[y|a,\theta]} = \frac{1}{a}.$$

I fix that parameter at 2%, which exogenously sets equal to 50 the target optimal values of a in program (1), denoted a^*. Changing the marginal effort productivity at the optimum to 10% ($a^* = 10$) or 1% ($a^* = 100$) does not affect the conclusions. Hence, I only report results for $a^* = 50$.

Next, the expected rate of return on assets, R, for instance, must be sensible. In their calibration with Compustat data for 10 industries, Boyd and Smith (1994) find that the annual gross rate of return on corporate assets from 1972 to 1991 ranged from 6.1% to 15.5%. I set $R = 12.1\%$, the nominal annual rate of return on common stocks during the 1926–1988 period (Brealey and Myers (1991)). In the robustness checks, values for R between 5% and 55% are also considered. With technology (6), the project's expected return is given by $E[y|a,\theta] - I = a\theta - I$. At the target optimal effort level under second-best financing, a^*, this implies that $\theta = (I \cdot R)/a^*$. Without loss of generality, I let $I = a^*$. Thus, for $a^* = 50$ and $R = 12.1\%$, $\theta = 1.121$ is obtained.

The coefficient of relative risk aversion does not have an accepted standard value but has a recognized range. In their study of the equity premium puzzle,

[9]In the Grossman and Hart (1983) framework used by Haubrich (1994), CEO effort increases the expected value of shareholder wealth. I restate managerial productivity in terms of firm value because, in that setup, equity holders are the firm's sole claimants and the manager's stake is small relative to other shareholders'.

[10]Alternatively, to avoid situations where managerial effort has a potentially infinite impact on expected cash flows, the action space could have been bounded away from 0 by exogenously imposing that the lowest possible managerial action be 1, thereby setting $E[y|a,\theta] = \theta$ under (6). Then, the productivity of effort could have been controlled by imposing that, at the optimal managerial action level under the calibration hypotheses, the expected firm value be larger than θ by a certain percentage—for example, 4% (Haubrich (1994)). In light of the results' robustness to various choices for the marginal effort productivity at the optimum, it is unlikely that this alternative setting would have changed the conclusions.

Mehra and Prescott (1985) specify (0,10] as a reasonable interval for economic agents who exhibit constant relative risk aversion. Under (5), however, the owner-manager has constant *absolute* risk aversion γ. Therefore, γ must be restricted so that, for likely levels of his wealth, the owner-manager's relative risk aversion remains in the appropriate range of (0, 10]. I let γ vary between 0.1 and 0.6. Given this range and given chosen values of θ and n, the owner-manager's relative risk aversion evaluated at his expected consumption, $\gamma(a^*\theta - I)$, runs from 2.63 to 6.45 in the first-best case, and from 1.41 to 5.46 in the second-best scenario. Values of γ lower than 0.1 imply very high rates of return on assets (more than 50%) in the second-best problem. Values of γ larger than 0.6 are not used, as they can imply average levels of relative risk aversion larger than 10 under equity financing.

Given all these parameter values, the degree of managerial disutility from effort, which is determined by n and A remains. The second parameter is a scaling factor. $A = I^2$ is set to obtain levels of disutility from effort in the same range as the expected utility from consumption. The first-order conditions of program (1) are used to calibrate n. For the parameter values chosen under (5) and (6), $n = 1.564$ when $\gamma = 0.5$. Accordingly, $n = 1.564$ is set as the central value for the computations below and takes values of n between 1.2 and 1.725 for robustness checks. Higher values of n are not used because in the second-best and, a fortiori, third-best environments, the manager strictly prefers not to work if $n > 1.725$—for all values of γ.

C. Scenarios

For tractability, the discussion now focuses on a few parameter combinations. The first or "most likely" scenario combines the parameter values just calibrated: $\theta = 1.121$; $n = 1.564$; $\gamma = 0.5$; and $I = 50$. The deadweight losses found for most of the parameter combinations listed in Table 1 are similar to the losses in this most likely case. In the second or "worst case" scenario, I select levels of managerial absolute risk aversion ($\gamma = 0.15$) and disutility from effort ($n = 1.2$) that I found yielded the largest deadweight costs from restricting financing to three traditional securities. The second-best rate of return on assets in this worst case scenario is quite high: 54.3%. Letting $\gamma = 0.1$ would not seriously affect the magnitude of the deadweight losses but would raise the rate of return on assets to an implausible 73.3%. Section VII introduces a third scenario to show that warrants are a key element of a traditional capital structure. This "warrant scenario" is identical to the most likely case except that managerial disutility from effort is very high ($n = 1.725$). I let $I = 50$ and $\theta = 1.121$ in all three scenarios because computations with other values of these two parameters showed that neither had a meaningful impact on my conclusions. Table 1 summarizes my parameter choices in the constant absolute risk-averse (CARA) specification.

TABLE 1

Parameter Choices

Parameter	Symbol	Most Likely Scenario	Worst Case Scenario	Warrant Scenario	Range Examined
Absolute risk aversion	γ	0.5	0.15	0.5	$0.1 \rightarrow 0.6$
Effort disutility	n	1.564	1.2	1.725	$1.2 \rightarrow 1.725$
Intrinsic asset productivity (% of assets)	$\theta - 1$	12.1%	12.1%	12.1%	5% → 55%
Marginal productivity of managerial effort	$1/a^*$	2%	2%	2%	1% → 10%
Firm size	I	50	50	50	$10 \rightarrow 100$

VI. Deadweight Costs of Restricting Financing to Debt, Equity, and Warrants

Using the parameters in Table 1, the first-best, second-best, and third-best decision variables, monetary payoffs, and expected managerial utility can be calculated. In each contracting environment, I compute certainty equivalent managerial consumption levels (CEC)—i.e., the consumption levels that would give the owner-manager the same utility with certainty. To quantify the deadweight losses brought about by various contracting constraints, the CEC in each environment are contrasted with the beginning-of-period assets (I). I also discuss how much further from their first-best levels the owner-manager's CEC and the rate of return on assets (ROA) fall when the basic agency problem is compounded by restrictions on admissible capital structures. These two comparisons provide a natural benchmark by which to assess the cost of more complex contracting, relative to the cost of contracting per se.[11] Table 2 summarizes the computations, while Figure 1 depicts the owner-manager's monetary payoffs.

A. Deadweight Losses from Moral Hazard under Optimal (Second-Best) Financing

Relative to the first-best environment, in which managerial effort levels are observable, the welfare losses brought about by incentive compatibility considerations are very substantial. In the most likely scenario, the owner-manager's certainty equivalent consumption falls by almost two-thirds because of his inability to precommit to the first-best effort level. That is, his CEC falls from 9.13 (18.3% of the assets I) in the first-best environment to 3.17 (6.3% of I) in the second-best environment. The deadweight loss reaches 27.6% of assets in the worst case scenario (his CEC drops from a first-best 33.3 to a second-best 19.5).

[11]Precisely, the losses brought about jointly by incentive compatibility requirements and by exogenous contracting constraints are compared to the losses induced solely by incentive compatibility requirements. By attributing the entire difference between the two deadweight cost levels to the exogenous contracting constraints, I implicitly assume that the incentive compatibility loss is independent from the contracting environment.

TABLE 2

Welfare Costs of Financial Contracting Restrictions ($\tau = 2$)

	Levels, in Units (expected managerial utility, CEC) or as Fraction of I (expected rate of return on assets)					Change from Second- to Third-Best, in % of Change from First- to Second-Best		
	First-Best	Second-Best (optimal)	Pure Equity	Equity and Debt	Third-Best (equity, debt, and warrants)	Pure Equity	Equity and Debt	Third-Best (equity, debt, and warrants)
Panel A. Most Likely Scenario								
CEC	9.13	3.17	2.54	3.00	3.16	−10.56	−2.85	−0.08
CEC/I	18.25%	6.33%	5.08%	5.99%	6.33%			
ROA	18.25%	12.10%	7.08%	12.63%	12.15%	−81.63	8.62	0.81
Panel B. Worst-Case Scenario								
CEC	33.31	19.54	18.37	18.37	19.47	−8.48	−8.48	−0.51
CEC/I	66.63%	39.08%	36.75%	36.75%	38.94%			
ROA	66.63%	54.30%	51.69%	51.69%	54.10%	−21.19	−21.19	−1.62
Panel C. Warrant Scenario								
CEC	7.69	0.99	0	0	0.97	−14.70	−14.70	−0.26
CEC/I	15.37%	1.97%	0	0	1.94%			
ROA	15.37%	5.37%	0	0	5.33%	−53.70	−53.70	−0.40

Managerial utility from consumption: $u(c) \equiv -e^{-\gamma c}$, with $\gamma = 0.5$ (Panels A and C) or $\gamma = 0.15$ (Panel B). Disutility from effort: $v(a) = a^n / I^2$, with $n = 1.564$ (Panel A); $n = 1.2$ (Panel B); or $n = 1.725$ (Panel C). Firm size: $I = 50$. Technology: $f(y, a) = 4y \exp(-2y/(a\theta))/(a\theta)^2$, with $\theta = 1.121$. For each contract, the table gives the rate of return on assets I (ROA) and the certainty equivalent managerial consumption level (CEC)—i.e., the consumption level that would give the owner-manager the same utility with certainty. No debt is issued in the worst case scenario unless warrants are bundled. The project is forgone in the warrant scenario under traditional financing unless warrants can be issued.

The large losses in these various scenarios are consistent with empirical estimates (Ferrall and Shearer (1994)) that moral hazard can be very onerous.

B. Deadweight Losses from Moral Hazard under Traditional Financing: Most Likely Scenario

The basic conjecture of this paper is that *simple* financing contracts, comprising a limited menu of traditional securities, can sufficiently differentiate the owner-manager's payoffs at high and low cash flow levels and, thus, can achieve welfare and efficiency properties very close to those of the highly non-linear second-best contract. The last column in Table 2 (Panel A) confirms this proposition by showing that the cost of limiting the securities floated to an optimally chosen combination of equity, one unsubordinated straight debt issue, and one warrant issue (with a single exercise price) is extremely small.

In the most likely scenario, the third-best financing constraints induce an additional CEC loss equal to only 0.08% of the loss caused by moral hazard itself. Likewise, imposing traditional financing yields a trivial change in the ROA from 12.1% to 12.15%. The reason for the mild ROA increase is that the owner-manager works slightly more under traditional financing than he would in the second-best environment.

To put the insignificance of these losses in perspective, I can express them in terms of the resources invested. The owner-manager's CEC falls by less than 0.01% of the investment I. This minuscule deadweight loss is representative of the trivial deadweight losses that can be computed with the vast majority of the parameter combinations listed in the last column of Table 1, which shows that the second-best (optimal) and third-best (traditional) outcomes are extremely close.

Indeed, suppose that the transaction costs associated with issuing or purchasing securities are higher for optimal than for traditional instruments. Further suppose that the transaction costs of implementing the optimal financing contract exceed those of using traditional securities by a mere 0.011% of the amount invested, I. Then, in the most likely scenario, the (third-best) combination of straight debt, equity, and warrants would strictly dominate the second-best contract in that the former would provide the owner-manager with the higher expected utility.

C. Deadweight Losses from Moral Hazard under Traditional Financing: Worst Case Scenario

When the owner-manager is neither very risk averse ($\gamma = 0.15$) nor very work averse ($n = 1.2$), his second-best monetary payoff is very sensitive to the project's cash flows once the latter exceed a certain target level—see Figure 1B. In the third-best environment, though, his payoff's performance sensitivity cannot exceed that achievable by floating only straight debt and warrants (adding external equity to the financing mix would decrease his monetary payoff's sensitivity to cash flows). As a result, the deadweight losses of imposing traditional financing are larger than for other reasonable parameter combinations in Table 1.

Yet, even in this worst case scenario, the third-best financing constraints bring about an extra CEC loss equal to a paltry 0.51% of the loss due to moral hazard only. In absolute terms, the owner-manager's CEC falls from a second-best 19.54 to a third-best 19.47. This loss amounts to less than 0.14% of the assets I (the first-best CEC was a massive 33.3).

Indeed, an optimal combination of a straight debt issue and a lone warrant issue would dominate the second-best financing contract as long as the cost of implementing the latter exceeds that of using these two traditional securities by just 0.12% of the amount invested, I. While that figure is 10 times larger than in the most likely scenario, it remains extremely small in comparison to the second-best ROA in this worst case scenario (54.3%).

In a series of papers, Amihud and Mendelson (1986), (1988), (1991) document that less liquid financial instruments have notably higher risk-adjusted rates of return.[12] Since markets for traditional securities are more liquid than those for customized financing vehicles, these authors argue that companies could signifi-

[12]Amihud and Mendelson (1986) present evidence that, for NYSE stocks, a 1% spread increase is associated with a 0.211% increase in *monthly* risk-adjusted excess returns. They also document (1991) that, after accounting for the spread and broker fees, the annual rate of return on Treasury bills is 0.388% lower than that on similar but less liquid Treasury notes.

cantly reduce their cost of capital by issuing traditional securities. One possible interpretation of the above results is that the cost of such an investment in liquidity would be minor: typically a negligible 0.01% of assets, and certainly less than 0.12%.

VII. Importance of Debt and Warrants

This study has shown that the deadweight loss from limiting the firm to equity, a straight debt issue, and a warrant issue is very small. This result, however, crucially depends on the ability to use all three of these securities. The warrant scenario in Table 2 helps illustrate this point. This case is identical to the most likely scenario except that managerial disutility from effort is extremely high ($n = 1.725$). The change makes moral hazard itself so costly that, even in the second-best environment, the owner-manager is almost indifferent between undertaking the project and not working (his CEC falls from a first-best 7.7 to only 0.98 in the second-best environment—see Table 2, Panel C). As a result, warrants become an essential component of the traditional capital structure.

A. Pure Equity Financing

Limiting external financing strictly to common stock (while granting the owner-manager a base salary and a residual equity stake) is not an innocuous restriction. It gives rise to large or very large deadweight losses over and above the losses caused by moral hazard. In the most likely scenario, for example, the owner-manager's CEC falls from 3.17 in the second-best environment to 2.54 under equity financing. This additional loss is more than 10% of the loss attributable to incentive compatibility problems, and is worth fully 1.25% of the investment I.

What is more important, for higher levels of managerial work aversion ($n \geq 1.6$), no investment takes place under equity financing even though it was optimal to invest in the second-best environment. The warrant scenario just introduced helps illustrate the intuition behind this result. When $n = 1.725$, the second-best contract awards the owner-manager nothing unless the project's cash flow exceeds a very high target level: $y = 51.1$ (see Figure 1C). In contrast, pure equity financing makes it impossible to both reward the owner-manager adequately when earnings are high and to punish him sufficiently for low cash flows. As a result, the owner-manager takes on the project if he can issue the second-best (optimal) securities—but not if he can issue equity only.

B. Debt and Equity Financing

Debt financing lessens the owner-manager's incentive to shirk, which allows investment to take place for many more parameter combinations than under pure equity financing. Indeed, with unsubordinated straight debt included in the financing mix, the project is forgone only for very high levels of managerial work aversion ($n \geq 1.7$) or when project quality (θ) and risk aversion (γ) are both very low.

Even when investment is already optimal under pure equity financing, adding debt to the choice of security offerings can significantly reduce the cost of exogenous contracting restrictions. In the most likely scenario, for example, restricting admissible financing instruments to just external equity and unsubordinated straight debt (while the owner-manager keeps a residual equity stake) decreases the owner-manager's CEC from 3.17 to 3. This number amounts to 0.34% of the assets I—much less than the 1.25% loss derived under pure equity financing.

The worst case scenario illustrates that adding straight debt to equity brings no benefit when the owner-manager is not very work averse ($n \leq 1.4$). His CEC drops from 19.54 in the second-best environment to 18.37 under equity financing: this decrease, equivalent to 2.33% of I, does not change when debt can be issued. The reason is that, with low managerial work aversion, ensuring incentive compatibility is easy and cash flows are plentiful. Consequently, the risk-averse owner-manager prefers a fixed compensation to an increase in his already large residual ownership of the company. The resulting combination of base salary and dividends provides sufficient motivation. Concurrently floating debt would be suboptimal. A corollary to this result is that the optimal equity and/or debt contract should never involve both external debt and a fixed salary for the owner-manager.

My analysis predicts that, given suitably chosen parameter values, any degree of leverage (defined as the ratio of payments promised to debtholders to the expected period-1 cash flow) can be optimal. Figure 2 shows that, ceteris paribus, leverage should decrease with project quality and increase with effort disutility. The latter result is qualitatively similar to Grossman and Hart's (1982) prediction that debt helps reduce managerial incentives to divert corporate resources away from investment in order to increase their perquisite consumption.

C. The Role of Warrants

I have shown that debt and equity financing causes a decrease in the owner-manager's CEC that is smaller than under pure equity financing yet remains non-negligible. Comparing this result with the findings of Section VI demonstrates the importance of including warrants in the class of admissible securities. In the most likely scenario, for instance, Table 2 shows that the CEC loss is equivalent to 0.34% of assets under debt and equity financing—but falls to less than 0.01% of assets once an issue of warrants with high enough exercise price is bundled with the bonds or outside equity. Likewise, the largest CEC loss computed in Section VI amounts to only 0.14% of assets. In contrast, the loss in this worst case scenario climbs to 2.33% of assets when warrants are banned. What is more important, the warrant scenario ($n = 1.725$) shows that cases exist where the owner-manager takes on the project only if he can float a warrant issue in addition to debt and equity.

The prediction that issuing warrants increases firm value is in line with empirical evidence, documented by Conrad (1989) and Detemple and Jorion (1990), that options are not redundant securities and that introducing individual options improves the risk-return tradeoff for the underlying stock. The result that bundling warrants together with straight debt can alleviate moral hazard is also consistent

FIGURE 1

Owner-Manager's Monetary Payoffs

Figure 1A. Most Likely Scenario

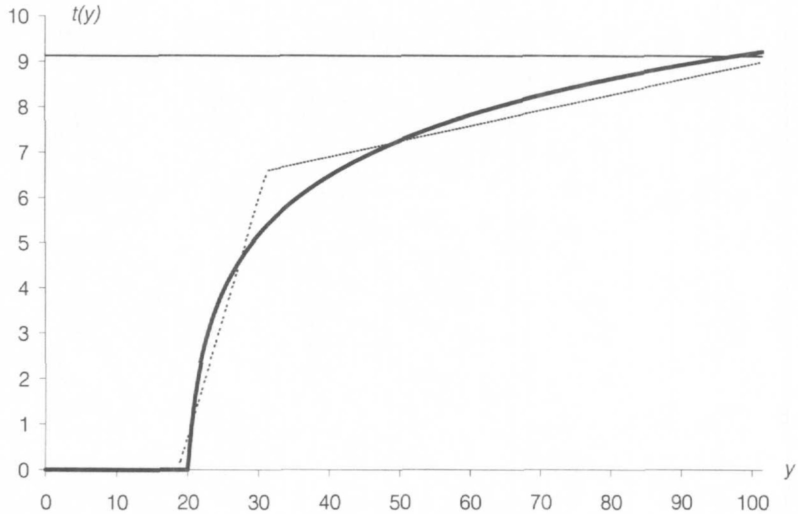

Figure 1B. Worst Case Scenario

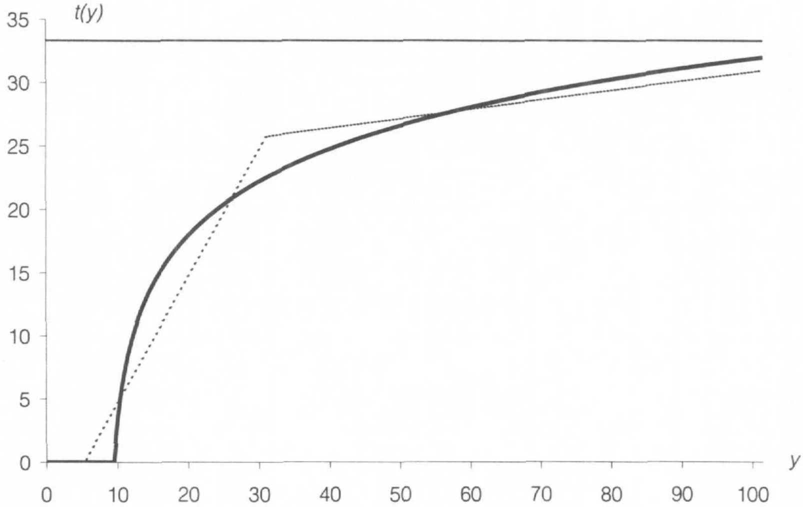

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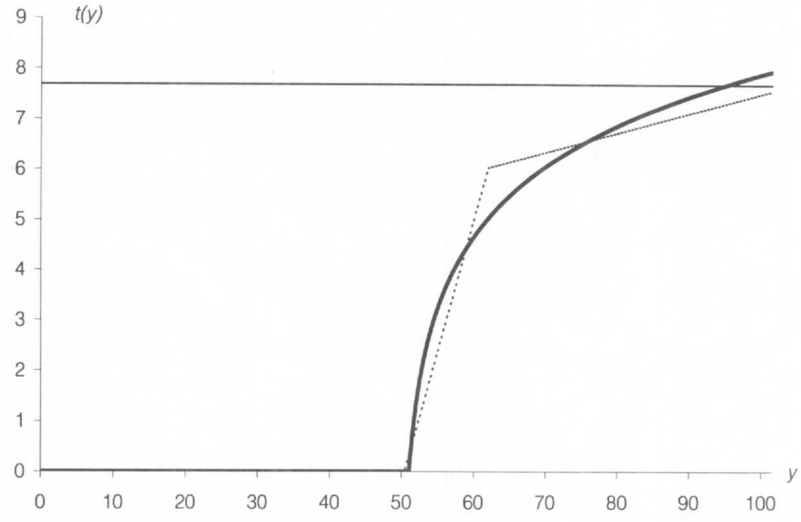
FIGURE 1 (continued)
Owner-Manager's Monetary Payoffs

Figure 1C. Warrant Scenario



Figures 1, A, B, and C: managerial preferences over consumption: $u(c) \equiv -e^{-\gamma c}$, with $\gamma = 0.5$ (Figures 1, A and C) or $\gamma = 0.15$ (Figure 1B). Disutility function from effort: $v(a) = a^n/I^2$, with $n = 1.564$ (1A); $n = 1.2$ (1B); or $n = 1.725$ (1C). Firm size: $I = 50$. Technology: $f(y, a) = 4y \exp(-2y/(a\theta))/(a\theta)^2$, with $\theta = 1.121$. The thin horizontal line gives the owner-manager's first-best monetary payoff. The solid logarithmic curve shows his second-best payoff $t^*(y)$ as a function of y. By construction, third-best monetary payoffs are continuous, three-piece piecewise-linear functions (shown in dashed line) of the cash-flow y. Equity, debt and one class of warrants are issued in the most likely (1A) and warrant (1C) scenarios, whereas only debt and warrants (but no external equity) are optimally issued in the worst-case scenario (1B).

with Green (1984) and Chiesa (1992), who posit risk-neutral managers. A basic difference with Green (1984) is that warrants here are a complement—not a substitute—to equity. In addition, my results generalize those of Chiesa (1992) by showing numerically that issuing a single class of warrants with the same exercise price suffices to obtain a nearly optimal financing package. In contrast, the optimal financing contract in Chiesa (1992) requires as many warrant issues with different exercise prices as there are states of nature.

VIII. Robustness

A. Parameters

A comparison of the three parts of Figure 1 illustrates that some of my results depend on the particular values chosen for the parameters. In the most likely scenario, for example, Figure 1A shows that the third-best contract optimally consists of a straight debt issue ($\alpha_1 = 0, X_1 > 0$), internal and external equity

FIGURE 2

Leverage under Debt and Equity Financing

Figure 2A

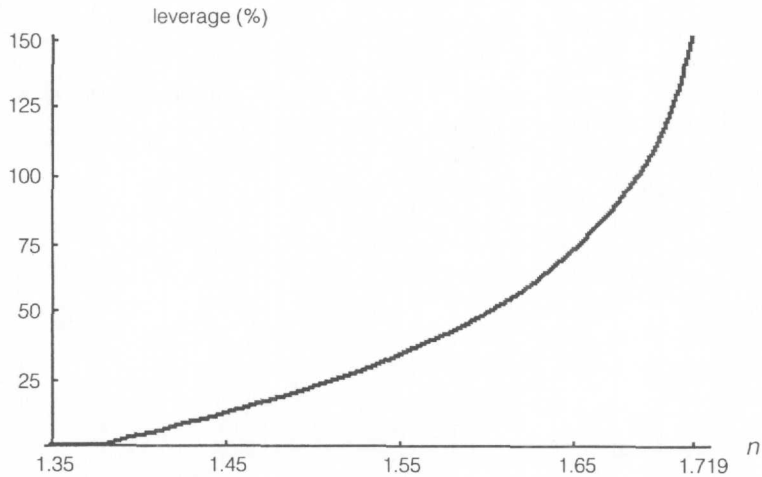
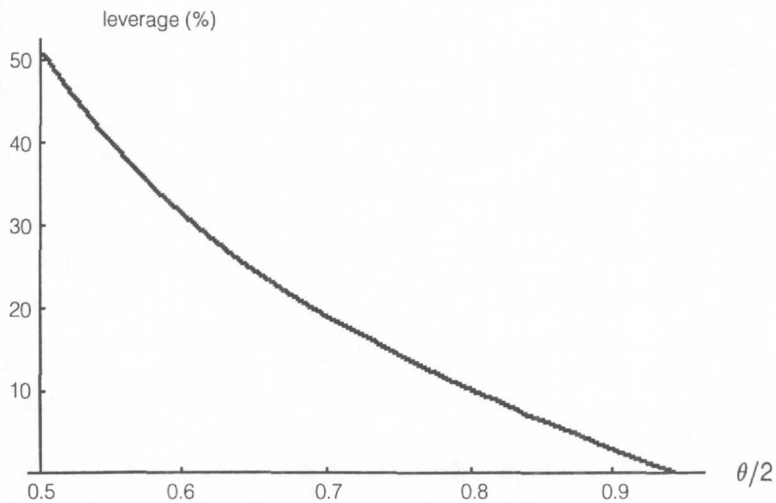


Figure 2B



Leverage is measured as the ratio of total repayments promised to debtholders to the expected period-1 cash flow $E[y] = a\theta$. Financing is limited to debt and equity—no warrants are allowed. I use the following parametrizations. Owner-manager's preferences over consumption: $u(c) \equiv -e^{-\gamma c}$, with coefficient of absolute managerial risk-version $\gamma = 0.5$. Technology: $f(y, a) = 4y\exp(-2y/(a\theta))/(a\theta)^2$, with intrinsic asset productivity θ. Power disutility from effort: $v(a) = a^n/I^2$, with coefficient of managerial disutility from effort n. Firm size: $I = 50$. Figure 2A sets $\theta = 1.121$ and shows how leverage changes as a function of n. Figure 2B sets $n = 1.564$ and plots how leverage varies as a function of the coefficient of intrinsic asset quality, θ.

$(0 < \alpha_2 < 1)$, and a warrant issue sold to outside investors $(\alpha_3 < \alpha_2)$. In contrast, Figure 1B shows that the third-best contract in the worst case scenario makes no use of external equity $(\alpha_2 = 1)$.

The main point of this paper, however, is not which traditional security is optimally part of the third-best package, but rather it is that some combination of equity and unsubordinated straight debt, together with a single warrant issue, is always nearly optimal. A comparison of columns 2 and 5 in Table 2 shows that this conclusion is robust to the choices of γ and n and to the expected rate of return on assets. The results are likewise robust to the project size, I, and to the value of the marginal managerial productivity at the optimum.[13]

B. Technology

The ability of traditional securities to closely approximate the optimal contract is also robust to alternative technology specifications. I first increased the shape parameter τ in distribution (6) from $\tau=2$ to $\tau=4$. At target values of a and θ, this change cut the variance (skewness) of the cash flow distribution by (more than) 50%. The cash flow distribution (6) was then replaced by the exponential distribution,

$$f(y,a) \equiv \frac{e^{-y/(a\theta)}}{a\theta}. \tag{8}$$

Technology (8) allows consideration of situations where the probability density of cash flows is monotone decreasing in the level of cash flows and most of the probability mass is concentrated on very low cash flow levels.

After re-calibrating the relevant parameter values, I found the conclusions unaffected by either technology change. Whether it is optimal to issue a given traditional security again depended on the parameter values employed. Furthermore, imposing the use of a limited menu of three traditional instruments led to deadweight losses as negligible as those found in Sections VI and VII.

C. Preferences

In all the parameterizations discussed thus far, it follows from condition (2) that the non-zero part of the owner-manager's second-best monetary payoff is concave in cash flows under CARA utility (5). It is natural to ask whether the costs associated with the exogenous contracting restrictions remain small when that payoff is convex in cash flows. I therefore replaced specification (5) by the alternative assumption that the owner-manager has constant relative risk-averse (CRRA) preferences over consumption and is not very risk averse,

$$u(c) \equiv \frac{c^{1-\sigma}-1}{1-\sigma}, \quad \frac{1}{2} < \sigma < 1. \tag{9}$$

In order to meet the Jewitt (1988) conditions, I set the relative risk aversion coefficient $\sigma > ½$ and re-calibrated the parameter values for two technologies: the

[13]Tables summarizing the robustness analyses discussed in this section are available upon request from the author.

gamma distribution (6) with $\tau = 2$, and the exponential distribution (8). In each case, the representative values chosen yield expected second-best ROAs around 31%—see Table 3. The latter number is high but seems acceptable for the smaller firms modeled.

TABLE 3

Welfare Costs of Financial Contracting Restrictions (CRRA Utility)

	Levels, in Units (expected managerial utility, CEC) or as Fraction of I (expected rate of return on assets)					Change from Second- to Third-Best, in % of Change from First- to Second-Best		
	First-Best	Second-Best (optimal)	Pure Equity	Equity and Debt	Third-Best (equity, debt, and warrants)	Pure Equity	Equity and Debt	Third-Best (equity, debt, and warrants)
Panel A. Scenario—Gamma Technology ($\tau = 2$)								
CEC	32.54	6.67	0	6.34	6.62	−25.76	−1.26	−0.19
CEC/I	65.09%	13.33%	0	12.68%	13.23%			
ROA	65.09%	31.47%	0	31.04%	31.40%	−93.63	−1.27	−0.21
Panel B. Scenario—Exponential Technology								
CEC	42.45	6.48	0	5.99	6.40	−18.00	−1.34	−0.21
CEC/I	84.90%	12.95%	0	11.98%	12.80%			
ROA	84.90%	31.65%	0	31.05%	31.56%	−59.44	−1.13	−0.16

Managerial preferences over consumption: $u(c) \equiv (c^{1-\sigma} - 1)/(1 - \sigma)$, with coefficient of relative managerial risk aversion $\sigma = 0.64$ (Panel A) or $\sigma = 0.625$ (Panel B). Disutility function from effort: $v(a) = a^n/I^2$, with $n = 2.1$ (Panel A), or $n = 2.15$ (Panel B). Firm size: $I = 50$. Technology: $f(y, a) = 4y \exp(-2y/(a\theta))/(a\theta)^2$ in Panel A; $f(y, a) = \exp(-y/(a\theta))/(a\theta)$ in Panel B, with $\theta = 1.121$. In both cases, the project is forgone under pure equity financing. For each contract, the table gives the rate of return on assets I (ROA) and the certainty equivalent managerial consumption level (CEC)—i.e., the consumption level that would give the owner-manager the same utility with certainty.

Table 3 shows that my results are broadly unaffected by the introduction of preferences (9). Banning all securities other than equity again creates large welfare losses, which the introduction of unsubordinated straight debt helps reduce. Moreover, forcing the owner-manager to only issue a few traditional instruments leads to very small deadweight losses as long as warrants can be used. Despite the high calibrated ROA, my third-best financing constraints make the owner-manager's CEC fall at most by 0.15% of the assets. This deadweight loss is directly comparable to that found in the worst case scenario (Table 2, Panel B). Under CRRA preferences (9), however, Figure 3 shows that the warrants are granted to the owner-manager rather than sold to outside investors.

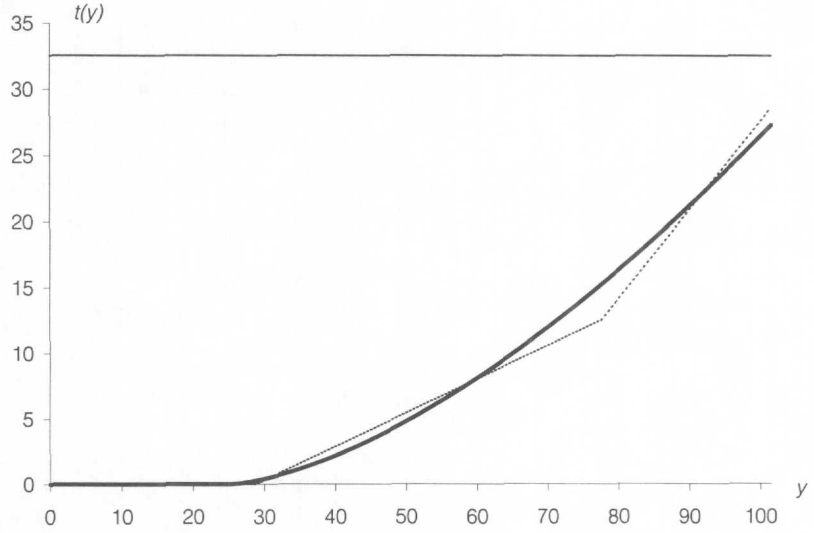
D. Discussion

Throughout this paper, I posit that the owner-manager has CARA or CRRA preferences over consumption. This choice is standard in the finance and agency

FIGURE 3

Owner-Manager's Monetary Payoffs: CRRA Utility

Figure 3A. Gamma Technology ($\tau = 2$)



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literatures,[14] which ensures that my conclusions do not result from atypical assumptions and are, therefore, directly comparable to the results derived by others. All parameterizations also satisfy the requirement that

$$(10) \qquad u\left(u'^{-1}\left(\frac{1}{z}\right)\right) \text{ is concave, } \quad z > 0.$$

This additional restriction on the curvature of the owner-manager's utility function is in line with the recent agency literature. In the absence of exogenous contracting restrictions, it presents the key advantage of validating the first-order approach discussed in Section III without imposing unreasonable limitations on the technology $f(y, a)$. The fact that the approach holds throughout the computations, despite various constraints on the admissible financing contracts, is in all likelihood a direct result of meeting condition (10).[15]

Jewitt (1988) argues that condition (10) is not very stringent. It does, admittedly, exclude owner-managers with extremely low constant relative risk aversion: $0 < \sigma \leq ½$. Ruling out such agents seems reasonable: for example, the lowest value used by Telmer (1993) or Heaton and Lucas (1996) to quantify the

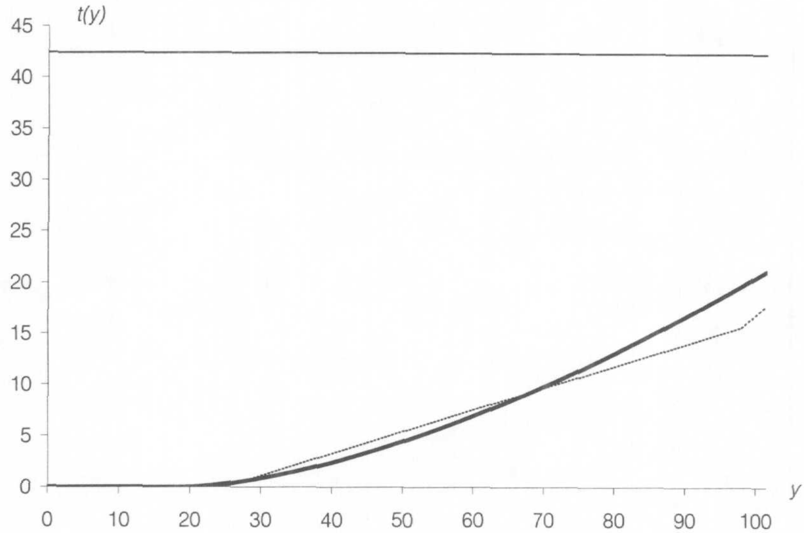
[14]For instance, all the agency and asset pricing papers discussed in Section II either make that assumption or posit risk neutrality.

[15]In practice, ignoring condition (10) means that the first-order approach may fail for some parameter combinations. With $\sigma = ½$, for example, this approach, in some cases, yielded a U-shaped payoff schedule $t(y)$ that was not incentive compatible.

FIGURE 3 (continued)

Owner-Manager's Monetary Payoffs: CRRA Utility

Figure 3B. Exponential Technology



In Figure 3, both A and B, managerial preferences over consumption: $u(c) \equiv (c^{1-\sigma} - 1)/(1-\sigma)$, with coefficient of relative managerial risk-version $\sigma = 0.64$ (Figure 3A) or $\sigma = 0.625$ (Figure 3B). Disutility function from effort: $v(a) = a^n/I^2$, with $n = 2.10$ (Figure 3A) or $n = 2.15$ (Figure 3B). Firm size: $I = 50$. Technology: $f(y, a) = 4y\exp(-2y/(a\theta))/(a\theta)^2$ in Figure 3A; or $f(y, a) = \exp(-y/(a\theta))/(a\theta)$ in Figure 3B, with $\theta = 1.121$. The thin horizontal line gives the owner-manager's first-best monetary payoff. The solid power curve shows his second-best monetary payoff as a function of the cash-flow y. By construction, his third-best monetary payoff (shown in dashed line) is a continuous, three-piece piecewise-linear function of y. Equity, debt and one class of warrants are issued in both cases. These warrants are given to the owner-manager; with the exponential technology, the warrants are not exercised unless y is very high.

importance of market completeness is $\sigma = 1.5$. Still, this seemingly innocuous assumption could contribute to the near optimality of standard securities in my setup.

To understand why, notice that agents whose preferences do satisfy condition (10) worry relatively more about their monetary payoff $t(y)$ at low levels of the cash flow y—see Dye (1986) and Jewitt (1988). Hence, across all parameterizations so far, the optimal financing contract must punish the owner-manager harshly when cash flows are low. As a result, given limited liability, the owner-manager's monetary payoff is identically zero (and, thus, trivially linear) under *both* optimal and traditional financings for the very cash flow levels that he cares most about.

To ascertain whether limited liability is instrumental to my results, two cases in which the owner-manager has very low relative risk aversion were first examined: $\sigma = ½$ and $\sigma = ⅓$. For these preferences, values of θ and n can be found such that managerial liability limits do not bind in the second-best contract if the ROA is sufficiently high. With $\sigma = ⅓$, Table 4 shows that the CEC decrease caused by issuing a combination of three securities (rather than optimal but non-traditional ones) exceeds 3.6% of the assets I with technology (6). The loss is even larger with the exponential technology (8), possibly because that probability distribution reinforces the impact of differences between the second-best and third-best monetary payoffs when y is low.

TABLE 4

Welfare Costs of Financial Contracting Restrictions CRRA Utility (Very High Managerial Risk Tolerance)

	Levels in Units (expected managerial utility, CEC) or as Fraction of I (expected rate of return on assets)				Change from Second- to Third-Best, in % of Change from First- to Second-Best	
	First-Best	Second-Best (optimal)	Equity and Warrants	Third-Best (equity, debt, and warrants)	Equity and Warrants	Third-Best (equity, debt, and warrants)
Panel A. CRRA $\sigma = ⅓$; Gamma Technology ($\tau = 2$)						
CEC	39.22	15.27	13.32	13.44	−8.10	−7.59
CEC/I	78.44%	30.53%	26.65%	26.89%		
ROA	78.44%	46.67%	42.48%	42.32%	−13.19	−13.69
Panel B. CRRA $\sigma = ⅓$; Exponential Technology						
CEC	48.15	12.19	10.08	10.33	−5.88	−5.19
CEC/I	96.30%	24.39%	20.16%	20.65%		
ROA	96.30%	45.25%	40.44%	40.39%	−9.43	−9.52
Panel C. CRRA $\sigma = ½$; Gamma Technology ($\tau = 2$)						
CEC	52.47	22.23	21.71	22.02	−1.70	−0.69
CEC/I	104.94%	44.45%	43.42%	44.04%		
ROA	104.94%	63.31%	62.18%	63.04%	−2.71	−0.66
Panel D. CRRA $\sigma = ½$; Exponential Technology						
CEC	54.92	12.94	12.30	12.72	−1.52	−0.53
CEC/I	109.85%	25.88%	24.60%	25.43%		
ROA	109.85%	49.14%	47.53%	49.02%	−2.66	−0.20

Managerial preferences over consumption: $u(c) \equiv (c^{1-\sigma} - 1)/(1-\sigma)$, with coefficient of relative risk aversion $\sigma = ⅓$ (Panels A, B) or $\sigma = ½$ (Panels C, D). Disutility function from effort: $v(a) = a^n/I^2$, with $n = 2.34$ (Panel A), $n = 2.30$ (Panel B), $n = 2.15$ (Panel C), or $n = 2.14$ (Panel D). Firm size: $I = 50$. The technology is the gamma distribution (6) with $\tau = 2$ in Panels A and C, or the exponential distribution (8) in Panels B and D. For all, $\theta = 1.121$. In all cases, the second-best contract is a very convex function of cash flows; as a result, investors' liability limit is binding in the second-best—i.e., $t^*(y) = y$ for all y greater than a given cash flow level. For each contract, the table gives the rate of return on assets I (ROA) and the certainty equivalent managerial consumption level (CEC)—i.e., the consumption level that would give the owner-manager the same utility with certainty.

These large costs might suggest that exogenous liability limits are indeed key to my results. Such is not the case, however. i) For CRRA preferences, Table 4 shows that the CEC loss from second-best to third-best falls to about 0.43% of the assets I once $\sigma = ½$—i.e., if managerial risk aversion is slightly higher than ⅓. Given that the second-best ROA ranges from 49% to 63%, this loss is small: indeed, the CEC loss over and above that caused by the existence of incentive compatibility problems is only between 0.53 and 0.69%—a deadweight cost similar to that found with CARA preferences. Furthermore, the CEC loss would fall to 0.25% of assets if the company were allowed to replace the unsubordinated straight debt issue by a second warrant issue with a different exercise price. ii) For CARA utility (5), deviations from limited liability correspond to negative managerial consumption—which is well defined for these preferences—and are optimal if allowed.[16] Yet, removing exogenous liability restrictions (condition (LL)) does not increase the gap between the solutions to the second-best program (1) and the third-best program (3). *Given* unlimited liability, the deadweight loss from traditional financing does not exceed 0.12% of I—a number similar to that computed in Section V under exogenous liability limits.

Figure 4 provides the clue as to why traditional financing can imply large deadweight losses when condition (10) is not met. The figure depicts the owner-manager's second-best and third-best monetary payoffs when he has extremely low risk aversion ($\sigma = 1/3, \sigma = 1/2$). When $\sigma = 1/3$, his second-best payoff is strongly convex in y.[17] Restricting financing to debt, equity and warrants, however, sharply limits the rate at which his monetary payoff can grow as cash flows increase (Figure 4A). Thus, there is a very large permanent difference between the second-best and third-best monetary payoffs once output exceeds a certain threshold—exactly those cash flow levels to which individuals with extremely high risk tolerance pay more attention.

In contrast, for customary preference specifications—those meeting condition (10)—major discrepancies between second-best and third-best monetary payoffs take place either over a limited interval of cash flow values (Figure 1B) or over a larger range of cash flow levels that the owner-manager is relatively less concerned about (Figure 3B). One can, therefore, expect that the near optimality of a very simple, traditional capital structure generalizes to all preference specifications for which the first-order approach holds.[18]

IX. Conclusions

Well-established firms raise external capital by selling traditional securities: debt, equity, and warrants. Yet, according to standard agency models, this type of financing is almost never optimal when managers can make a significant contribution to their firm's performance, have different objectives than outside investors, and hold a significant stake in the company.

[16] This point is made in Robe (1997). Using a framework similar to the second-best here, that paper shows limited liability must bring about large welfare gains for the institution to be optimal under moral hazard.

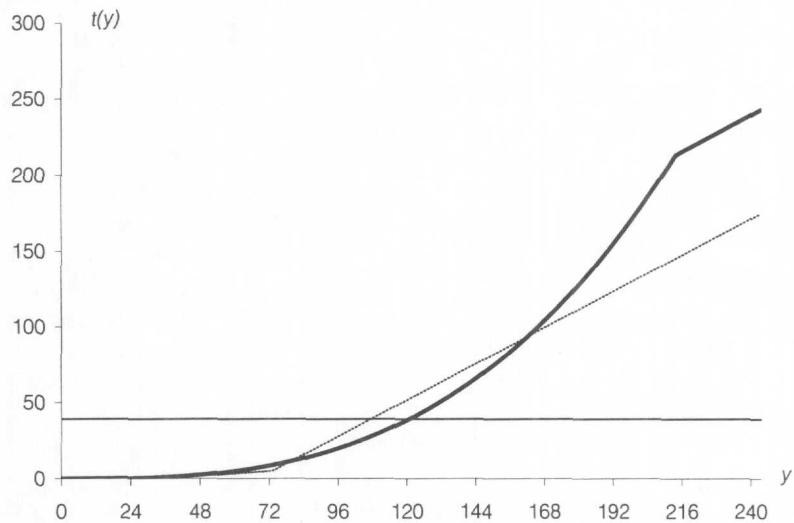
[17] With most cash flow distributions, it is precisely because the convexity of $t^*(y)$ is strong enough to overcome the agent's weak risk aversion that the first-order approach fails for at least some parameter combinations.

[18] Mathematically, because of the stationary character of the variational principle (the objective function in Program (1) is maximized at the exact $t^*(y)$ from Euler equation (2)), it is expected that one can replace $t^*(y)$ by a fairly crude approximation (my debt, equity, and warrants combination) and not significantly affect the value of the objective function at the optimum. In that sense, replacing optimal (but non-traditional) securities by a trio of traditional instruments is remindful of the Rayleigh-Ritz method of successive approximation—see Morse and Feshbach ((1953), pp. 1106-1119).

FIGURE 4

Owner-Manager's Monetary Payoffs CRRA Utility
(Very High Managerial Risk Tolerance)

Figure 4A. CRRA $\sigma = ⅓$, Gamma Technology ($\tau = 2$)



(continued on next page)

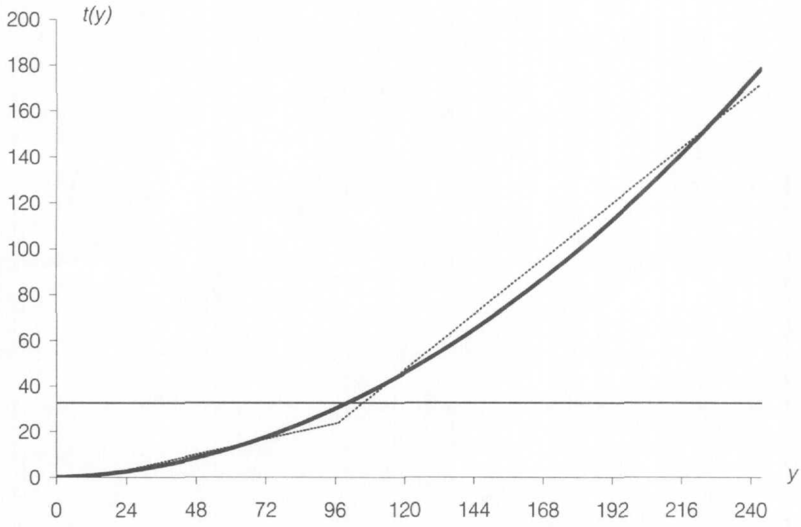
This paper quantifies the deadweight costs borne by the issuer as a result of only using equity, debt, and warrants in a standard moral hazard model where the optimal financial contract is likely to be very complicated. Given that moral hazard itself is very costly, I show numerically that the extra deadweight loss from restricting security offerings to equity, one straight debt issue (without multiple layers of seniority), and one warrant issue (without multiple exercise prices) is extremely small.

Barring all securities other than equity does cause very large additional losses. With unsubordinated straight debt included in the financing menu, the losses are smaller than under pure equity financing but remain non-negligible. The paper's second major result is that the addition of a single warrant issue (with a unique exercise price) is enough to cut the deadweight losses dramatically. Indeed, if the transaction costs of implementing the optimal financing contract exceeded those of using this trio of traditional securities by just 0.011% (0.15%) of the amount invested, then a combination of equity, one straight debt issue, and one warrant issue would become the optimal contract for most (all) parameterizations. The robustness of this conclusion to a wide range of common parameterizations suggests that the gains from adopting non-standard financing contracts are, in practice, likely to be minor.

FIGURE 4 (continued)

Owner-Manager's Monetary Payoffs CRRA Utility
(Very High Managerial Risk Tolerance)

Figure 4B. CRRA $\sigma = ½$, Gamma Technology ($\tau = 2$)



In Figure 4, both A and B, managerial preferences over consumption: $u(c) \equiv (c^{1-\sigma} - 1)/(1-\sigma)$, with coefficient of relative managerial risk-version $\sigma = ⅓$ (Figure 4A) or $\sigma = ½$ (Figure 4B). Disutility function from effort: $v(a) = a^n/I^2$, with $n = 2.34$ (4A) or $n = 2.15$ (4B). Firm size: $I = 50$. Technology: $f(y,a) = 4y\exp(-2y/(a\theta))/(a\theta)^2$ with $\theta = 1.121$ in both cases. The thin horizontal line gives the owner-manager's first-best monetary payoff. The solid power curve shows his second-best monetary payoff $t^*(y)$ as a function of the cash-flow y; in both cases, $t^*(y)$ is strictly positive for all y. In Figure 4A, investors' limited liability constraint is binding (i.e., $t^*(y) = y$) when $y > 213.76$. By construction, the owner-manager's third-best monetary payoffs (shown in dashed line) are continuous, three-piece piecewise-linear functions of y. Equity, debt and a single class of warrants (given to the owner-manager) are issued in both cases. With $\sigma = ⅓$, there is a massive discrepancy between the second-best and third-best monetary payoffs for high values of the cash-flow y.

Appendix: Functional Form of the Second-Best Managerial Payoff

When positive, the second-best monetary payoff $t^*(y)$ must satisfy the Euler equation,

$$\frac{1}{u'(t(y))} = \overline{\lambda} + \overline{\mu}\frac{f_a(y,a)}{f(y,a)} \qquad \text{for a.e. } y.$$

Substituting CARA utility (5) and the gamma technology (6) into this Euler equation yields

$$t(y) = \frac{1}{\gamma}\ln\left[\overline{K} + \overline{H}y\right] \qquad \text{where} \qquad \overline{K} \equiv \frac{\gamma}{\lambda}\left[1 - \frac{\mu\tau}{a}\right]$$

$$\text{and} \qquad \overline{H} \equiv \frac{\gamma\mu\tau}{\lambda\alpha^2\theta}.$$

Section VIII considers a situation in which the gamma technology (6) is unchanged, but the manager has CRRA preferences (9). In this third parameterization, the Euler equation yields,

$$t(y) = \left[\overline{K} + \overline{H}y\right]^{\frac{1}{\sigma}} \qquad \text{where} \qquad \overline{K} \equiv \frac{1}{\lambda}\left[1 - \frac{\mu\tau}{a}\right]$$

$$\text{and} \qquad \overline{H} \equiv \frac{\mu\tau}{\lambda a^2\theta}.$$

References

Allen, F., and G. Gale. "Optimal Security Design." *Review of Financial Studies,* 1 (1988), 229–263.

________________. *Financial Innovation and Risk Sharing.* Cambridge, MA: The M.I.T. Press (1994).

Allen, F., and A. Winton. "Corporate Financial Structure, Incentives and Optimal Contracting." Working Paper, Rodney L. White Center for Financial Research (June 1994).

Amihud, Y., and H. Mendelson. "Asset Pricing and the Bid-Ask Spread." *Journal of Financial Economics,* 17 (1986), 223–249.

________________. "Liquidity and Asset Pricing: Financial Management Implications." *Financial Management,* 17 (1988), 5–15.

________________. "Liquidity, Maturity, and the Yields on U.S. Treasury Securities." *Journal of Finance,* 46 (1991), 1411–1425.

Baxter, M., and M. Crucini. "Business Cycles and the Asset Structure of Foreign Trade." *International Economic Review,* 36 (1995), 821–854.

Bhattacharya, S., and P. Pfleiderer. "Delegated Portfolio Management." *Journal of Economic Theory,* 36 (1985), 1–25.

Boyd, J., and B. Smith. "How Good are Standard Debt Contracts? Stochastic vs. Non-Stochastic Monitoring in a Costly State Verification Environment." *Journal of Business,* 67 (1994), 539–561.

Brealey, R., and S. Myers. *Principles of Corporate Finance,* (4th ed.) New York, NY: McGraw Hill (1991).

Chiesa, G. "Debt and Warrants: Agency Problems and Mechanism Design." *Journal of Financial Intermediation,* 2 (1992), 237–254.

Conrad, J. "The Price Effect of Option Introduction." *Journal of Finance,* 44 (1989), 487–498.

Detemple, J., and P. Jorion. "Option Listing and Stock Returns: An Empirical Analysis." *Journal of Banking and Finance,* 14 (1990), 781–801.

Dybvig, P., and C. Spatt. "Agency and the Market for Mutual Fund Managers: The Principle of Preference Similarity." Working Paper, Carnegie Mellon and Yale Univ. (March 1986).

Dye, R. A. "Optimal Monitoring Policies in Agencies." *Rand Journal of Economics,* 17 (1986), 339–350.

Ferrall, C., and B. Shearer. "Incentives, Team Production, Transactions Costs and the Optimal Contract: Estimation of an Agency Model Using Payroll Records." Working Paper, Queen's Univ. (Aug. 1994).

Fluck, Z. "Optimal Financial Contracting: Debt versus Outside Equity." *Review of Financial Studies,* 11 (1998), 383–418.

Gale, D. "Standard Securities." *Review of Economic Studies,* 59 (1992), 731–755.

Gjesdal, F. "Piecewise Linear Incentive Schemes." *Scandinavian Journal of Economics,* 90 (1988), 305–328.

Green, R. C. "Investment Incentives, Debt, and Warrants." *Journal of Financial Economics,* 13 (1984), 115–136.

Grossman, S., and O. Hart. "Corporate Financial Structure and Managerial Incentives." In *The Economics of Information and Uncertainty,* J. McCall, ed. Chicago, IL: Chicago Univ. Press (1982).

________________. "An Analysis of the Principal Agent Problem." *Econometrica,* 51 (1983), 7–46.

Harris, M., and A. Raviv. "Optimal Incentive Contracts with Imperfect Information." *Journal of Economic Theory,* 20 (1979), 231–259.

Haubrich, J. "Risk Aversion, Performance Pay, and the Principal-Agent Problem." *Journal of Political Economy,* 102 (1994), 258–276.

Heaton, J., and D. Lucas. "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing." *Journal of Political Economy,* 104 (1996), 443–487.
Helwege, J., and N. Liang. "Is There a Pecking Order? Evidence from a Panel of IPO Firms." *Journal of Financial Economics,* 40 (1996), 429–458.
Holmström, B. "Moral Hazard and Observability." *Bell Journal of Economics,* 10 (1979), 74–91.
Innes, R. "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory,* 52 (1990), 45–67.
Jewitt, I. "Justifying the First-Order Approach to Principal Agent Problems." *Econometrica,* 56 (1988), 1177–1190.
Mehra, R., and E. C. Prescott. "The Equity Premium: A Puzzle?" *Journal of Monetary Economics,* 15 (1985), 145–161.
Morse, P., and H. Feshbach. *Methods of Theoretical Physics - 1.* New York, NY: McGraw Hill (1953).
Petersen, M., and R. Rajan. "The Benefits of Lending Relationships: Evidence from Small Business Data." *Journal of Finance,* 49 (1994), 3–37.
Robe, M. "How Costly are Limited Liability Rules?" Working Paper MR-02, Finance Research Center, McGill Univ. (Nov. 1998).
Ross, S. "On the Economic Theory of Agency and the Principle of Similarity." In *Essays on Economic Behavior under Uncertainty,* Balch, McFadden, and Wu, eds. North Holland (1974).
Royden, H. L. *Real Analysis,* 3rd ed. New York, NY: Macmillan (1988).
Stoughton, N. "Moral Hazard and the Portfolio Management Problem." *Journal of Finance,* 48 (1993), 2009–2028.
Sung, J. "Linearity with Project Selection and Controllable Diffusion Rate in Continuous-Time Principal-Agent Problems." *Rand Journal of Economics,* 26 (1995), 720–743.
Telmer, C. I. "Asset Pricing Puzzles and Incomplete Markets." *Journal of Finance,* 48 (1993), 1803–1832.
Weisbach, M. "Outside Directors and CEO Turnover." *Journal of Financial Economics,* 20 (1988), 431–460.
Welch, K. D. "Bankruptcy, Debtor's Prison, and Collateral in Optimal Debt Contracts." Working Paper, Univ. of Chicago (Dec. 1995).
Williams, J. "Debt and Equity as Optimal Financial Contracts." Working Paper, Univ. of British Columbia (Nov. 1988).